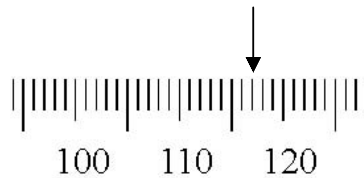
# Significant Figures and Experimental Uncertainty

No *measurement* that we can ever make is perfect or EXACT. All measurements have a certain amount of UNCERTAINTY associated with them. Only when we COUNT a set of objects do we get an EXACT number (e.g., 3 cabbages, 25 pencils).

I. Significant Figures

The *number of significant figures* in a measurement is defined to be all of the certain figures PLUS the *first uncertain* figure. In other words, we say all the reasonably reliable figures are significant figures.

**Example:** In the figure at the right, the measured value (indicated by the arrow) has 3 certain figures (i.e., 1, 1 and 7) and the 1 uncertain figure (i.e., 3, or is it a 2? or a 4?) In other words, the “3” is *uncertain*, but we are reasonably sure that the actual value of the fourth digit is *close* to 3.




If one is *given* a measured value, then THE NUMBER OF SIGNIFICANT FIGURES IS DETERMINED BY WRITING THE VALUE IN SCIENTIFIC NOTATION AND COUNTING THE DIGITS. *We always assume* the last digits is somewhat uncertain.

**Example:**

117.3 = 1.173 X $10^2$, and 1.173 has 4 significant figures
0.008 51 = 8.51 X $10^{-3}$ and 8.51 has 3 significant figures

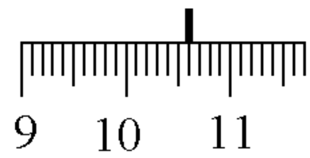
*Note* that the two zeros after the decimal in 0.00851 are not significant. We do not start counting digits until we encounter the ***first non-zero digit*** (starting from the left), and once we start to count we continue until all the digits to the right of the first significant digit have been counted.

**Example:** 0.003 000 0 = 3.0000 X $10^{-3}$ has 5 significant digits since all digits to the right of the first significant digit, the 3 must be counted.

The reason why the zeroes in 0.008 51 are not significant can be easily seen. Suppose a piece of metal was 8.51mm long. We can all agree that the length is known only to 3 significant figures. If we are asked to express this length in metres, rather than millimetre, the length would be 0.008 51 m. We still have the same piece of metal and the same uncertainty in the last figure. It is not reasonable to now insist that we know the length to 5 significant figures (*i.e.*, one hundred times more accurately), since we have only changed the units and have not actually made a more accurate measurement.

## II. Uncertainty

We define the uncertainty of a number to be the approximate amount by which the last significant digit *might* be in error.




**Example:** In the figure at the right, the pointer seems to be centered on a value of 10.60, but since the pointer is so wide the value might be as high as 10.62 or as low as 10.58.
Therefore we say the value is 10.60 ±0.02.

*Note* that when we say a value is 15.3 ±0.6, we mean the value could be as low as 15.3 – 0.6 = 14.7 and as high as 15.3 + 0.6 = 15.9

**Example:** The value for a measurement could be as high as 15.6 and as low as 15.2. What value and uncertainty for the measurement should be recorded?

Since the values 15.2 and 15.6 represent the minimum and maximum possible values, we will use the value in the *middle* of these two values for the number we record:

$$\text{average} = \frac{15.2 + 15.6}{2} = 15.4$$

Now, the uncertainty is just the *difference* between the largest value and the average (or between the average and the smallest value):

$$\text{Uncertainty} = 15.6 - 15.4 = 0.2$$

Hence we would record: 15.4 ±0.2

**Example:** The values for a measurement could be as high as 0.064 and as low as 0.058. What value and uncertainty for the measurement should be recorded?

$$\text{average} = \frac{0.063 + 0.058}{2} = 0.0605$$

$$\text{uncertainty} = 0.063 - 0.0605 = 0.0025$$

But our original measurement contained enough uncertainty that we have to accept any number between 0.058 and 0.063 (only 2 significant figures). Hence, to claim that the average has 3 significant figures is a little silly. We therefore round off both the average and the uncertainty:

$$\text{Value recorded} = 0.061 \pm 0.003$$

For the ruler, thermometer and graduated cylinder the *uncertainty* listed is 1/5 of the smallest division on the scale to be read, and the *alternate uncertainty* is ½ of the smallest division. Balance uncertainties are “built-in”; balances cannot detect any mass difference smaller than those listed (unless you get a more accurate balance). Time uncertainties depend on human reflexes.

## III. Rounding Off

When we have a number with more digits than we require, we must ROUND OFF the number.

We shall define "*last digit*" to be the last of the significant figures in which we are interested, such that we are going to get rid of all digits to the right of the "*last digit*". We shall also define "*deciding digit*" to be the digit to the immediate right of the "*last digit*", and "*successive digits*" to be all the digits (if any) to the right of the deciding digit.

**Example:** We wish to round off 5.237 5 to 3 significant figures.

Last Digit Deciding Digit

The procedure for rounding off a number is as follows:

i. If the deciding digit is less than 5, we dimply OMIT the deciding digit and successive digits
ii. if the deciding digit is 5 or greater, we drop the deciding and successive digits and increase the last digit by 1

4.124 rounded off to 3 significant figures is 4.12
0.0712 rounded off to 1 significant figure is 0.07
0.414 99 rounded off to 2 significant figures is 0.41

4.126 rounded off to 3 significant figures is 4.13
0.0351 rounded off to 1 significant figure is 0.04

## IV. How Many Significant Figures In An Aswer?

### A. Multiplying and Dividing

The answer to a multiplication or division should have only as many significant figures as the *least accurate number used* (the number having the least number of significant figures).

**Example:**

| 4.1 | X | 5.67 | = | 23 |
|---|---|---|---|---|
| (2 sig figs) | | (3 sig figs) | | (2 sig figs) |

The answer is limited to 2 significant figures because the smallest number of significant figures involved in the multiplication was 2.

**Example:**

| 12 975.6 | X | 423.4 | = | $5.494 \times 10^6$ |
|---|---|---|---|---|
| (6 sig figs) | | (4 sig figs) | | (4 sig figs) |

We use scientific notation to show that the answer has only 4 significant figures. An answer of 5 494 000 could mean 4, 5, 6 or 7 significant figures.

B. Adding and Subtracting

The number of significant figures in the answer to an addition or subtraction problem is limited by the *smallest* number of digits ***after*** the decimal point.

**Examples:** The answers are *underlined* given to the correct number of significant figures.

| | | |
|---|---|---|
| 123.45 | 123.42 | 123.456 |
| + 51.62 | + 53.2 | + 0.3 |
| 175.07 | 176.62 | 123.765 |
| | 176.2 | 123.8 |

| | | |
|---|---|---|
| 123.45 | 123.45 | 123.476 |
| - 61.32 | - 51.3 | - 123.476 |
| 62.13 | 72.12 | 0.002 |
| | 72.1 | |

V. When Should I Round Off?

You should ***always*** avoid rounding off your ***intermediate*** results; only the final answer should be rounded off. To avoid carrying an excessive number of digits keep 1 extra digit as you carry answers through a calculation.

**EXERCISES**

1. State the number of significant figures in:

a) 3 570 b) 17.505
c) 41.400 d) 0.51
e) 0.000 573 f) 0.009 00
g) $41.50 \times 10^{-4}$ h) $0.007\ 160 \times 10^{5}$
i) $123\ 400 \times 10^{-8}$ j) $0.000\ 410\ 0 \times 10^{7}$

2. Round off:

a) 4.954 93 to 5 sig. figs. b) 4.954 93 to 4 sig. figs.
c) 4.954 93 to 3 sig. figs. d) 4.954 93 to 2 sig. figs.
e) 0.005 06 to 2 sig. figs. f) 95 147.2 to 3 sig. figs.
g) 0.449 99 to 1 sig fig. h) 0.005 945 to 3 sig. figs.

3. Re-write using scientific notation, to indicate 3 significant figures (round off if necessary)

a) 35 700 b) 0.005 167
c) 45.05 d) 175 400.4
e) -0.547 36 f) 0.009 00

4. Perform the indicated operations and give the answer to the correct number of significant figures.

a) 12.5 * 0.50
b) 0.15 * 0.001 6
c) 40.0 * 30.000 0
d) 2.5 * 7.500 / 0.150
e) (6.40 X $10^{8}$) * (5 X $10^{5}$)
f) 4.37 X $10^{3}$ / 0.008 560 0
g) 51.3 * 3.940
h) 0.51 X $10^{-4}$ / 6 X $10^{-7}$
i) 4.75 * 5
j) 0.000 00 / 0.100 0
k) 7.4 * 3
l) 0.000 43 * 0.005 001

5. Perform the indicated operations and give the answer to the correct number of significant figures.

a)
  7.5
+ 4.6

b)
  14.35
+ 7.6

c)
  9.40
- 9.306

d)
  19.045
- 19.044

e)
  0.005 16
+ 0.317

f)
  4.1378 X$10^{4}$
- 7.569 X$10^{2}$

g)
  0.0075 X $10^{3}$
+ 5.1376 X $10^{2}$

h)
  7.63 X$10^{4}$
+ 3.4 X$10^{2}$